TABLE IV
EIGENVALUES (k_0, cm^{-1}) FOR AN EMPTY SPHERICAL CAVITY OF RADIUS 1 cm

Mode	Analytical	Computed 300 Unknowns	Error (%)
TM_{010}	2.744	2.799	−2.04
$TM_{111,even}$		2.802	−2.11
$TM_{111,odd}$		2.811	−2.44
TM_{021}	3.870	3.948	−2.02
$TM_{121,even}$		3.986	−2.99
$TM_{121,odd}$		3.994	−3.20
$TM_{221,even}$		4.038	−4.34
$TM_{221,odd}$		4.048	−4.59
TE_{011}	4.493	4.433	1.33
$TE_{111,even}$		4.472	.47
$TE_{111,odd}$		4.549	−1.25

TABLE V
TEN LOWEST NON-TRIVIAL EIGENVALUES (k_0, cm^{-1}) FOR THE GEOMETRY DRAWN IN FIG 2: (a) 267 UNKNOWNS; (b) 671 UNKNOWNS

No.	(a)	(b)
1	4.941	4.999
2	7.284	7.354
3	7.691	7.832
4	7.855	7.942
5	8.016	7.959
6	8.593	8.650
7	8.906	8.916
8	9.163	9.103
9	9.679	9.757
10	9.837	9.927

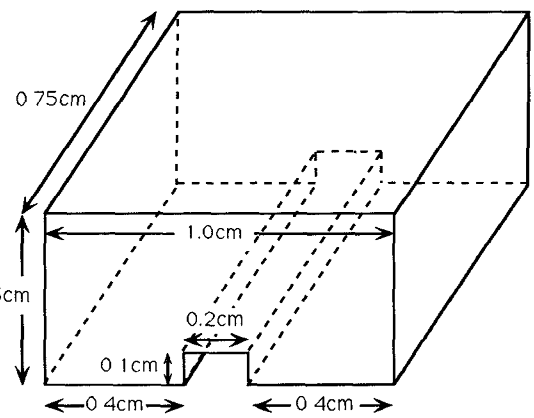

Fig. 2. Geometry for Table V.

ment solution to within 1 percent (no symmetry was assumed in this solution). Similar comparisons are given in Table IV for a sphere having 1 cm radius. Finally, Table V presents the eigenvalues of the geometry illustrated in Fig. 2. This is a closed metallic cavity with a ridge along one of its faces.

It is noted that as the degeneracy of the eigenvalues increases, the matrix becomes increasingly ill-conditioned and the numerical solution is correspondingly less accurate [10]. This is clearly observed from the data in Table IV for the case of a perfectly conducting hollow spherical cavity. Since the second lowest TM mode has five-fold degeneracy, the computational error is seen to be the greatest. However, for the partially filled rectangular cavity, the absence of degenerate modes gives results which are accurate to within 1 percent of the exact eigensolutions. We finally remark of the inherent presence of zero eigenvalues in our computations whose number is equal to the internal nodes. These zero eigenvalues are easily identifiable and since they do not correspond to physical modes, they were always discarded.

IV. CONCLUSIONS

It was shown that the resonant frequencies of an arbitrarily shaped inhomogeneously filled metallic resonator can be computed very accurately via the finite element method using edge-based tetrahedral elements. The same method in conjunction with node-based elements is much less reliable and not readily applicable to regions containing discontinuous boundaries in shape and material. Edge-based rectangular bricks do not provide as good an accuracy as edge-based tetrahedral elements and their use is further limited to a special class of geometries.

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Dielectric Property Measurements of Materials Using the Cavity Technique

Ahmet Baysar and James L. Kuester

***Abstract*—A cavity technique based on frequency shift was used to measure dielectric properties (dielectric constant and loss factor) of some particulate materials as a function of temperature. The materials studied were alumina, cobalt/alumina, dolomite and sand. The prop-**

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The authors are with the Department of Chemical, Bio and Materials Engineering, Arizona State University, Tempe, AZ 85287.
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erties were measured at various points between room temperature and 610°C in the frequency range of 925–995 MHz. The dielectric constant and loss factor of all samples, except the cobalt/alumina catalyst sample, were approximately constant with temperature. The dielectric constant and loss factor of the cobalt/alumina sample exhibited a noticeable increase with temperature.

Introduction

In a recent article [1], the dielectric properties of high purity polycrystalline silicon at various temperatures and particle size were reported. The knowledge of dielectric properties (dielectric constant and loss factor) is essential for predicting heating efficiency and behavior of materials processed using microwaves. Dielectric properties at elevated temperatures are not generally available in the literature. In this paper, the dielectric properties of several materials of interest are reported.

Experimental Method and Results

A rectangular cavity operating in the TE_{101} mode was used between room temperature and 610°C. The materials studied had relatively low losses. Therefore, the material losses were not included in the theoretical analysis.

Consider the rectangular cavity shown in Fig. 1. Region 1 is filled with air while region 2 contains the dielectric material. At resonance, it can be shown that the sum of impedances looking to the left, Z_1 and to the right, Z_2, at the air-dielectric interface is zero [2]. Then, the following equation may be derived for the partially filled cavity:

$$\frac{\tan(\beta_1 l_o)}{\beta_1} = -\frac{\tan(\beta_1 l_e)}{\beta_2} \tag{1}$$

where

β_1 = phase constant in region 1,
β_2 = phase constant in region 2,
l_e = length of the dielectric material
and l_o = length of the air-filled region 1.

The phase constants in both regions are given by

$$\beta_1 = \sqrt{\omega_r^2 \mu_o \epsilon_o - \left(\frac{\pi}{a}\right)^2} \tag{2}$$

and

$$\beta_2 = \sqrt{\omega_r^2 \mu_o \epsilon_o \epsilon' - \left(\frac{\pi}{a}\right)^2} \tag{3}$$

where

ω_r = angular TE_{101} mode resonant frequency of the partially filled cavity,
μ_o = permeability of free space,
ϵ_o = dielectric constant of free space,
ϵ' = relative dielectric constant of the dielectric material,
and a = length of the wide dimension of the cavity.

The values for all parameters in (1)–(3), except ϵ' and β_2, are known. Equation (1) is a transcendental equation and solution for ϵ' must be obtained by numerical means. The loss factor is calculated using the cavity technique analysis [1]. Bhartia and Hamid [3], reported dielectric measurements for sheet materials using a shorted rectangular waveguide. In this study, the dielectric property measurement of particulate materials in a rectangular cavity with temperature is reported.

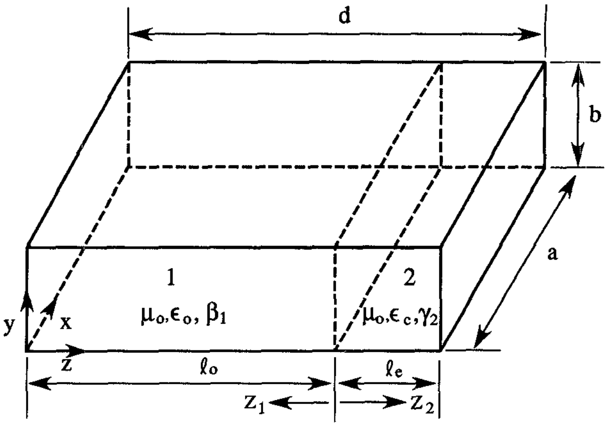

Fig. 1. Rectangular cavity partially loaded with dielectric.

The stainless steel cavity shown in Fig. 2, when empty, had a TE_{101} mode resonant frequency of 995.2 ± 1 MHz. The dimensions of the rectangular cavity were 24.8 cm × 12.38 cm × 18.96 cm. A thermocouple was intruded 0.8 cm inside the cavity through the center of the bottom plate for minimum electromagnetic field disturbance. A removable top plate permitted loading and leveling of particulate samples. All samples were in particulate form ranging from 40 to 270 mesh (53–425 μm). The sample leveling piece, shown on the right side of Fig. 2, was used to level the sample in the cavity so that uniform sample thickness was achieved from corner to corner. The upper piece of the leveler could be moved up or down using the set screws to adjust the length of the leveler edge (l_o, the depth of air-filled region). Microwave signal was coupled into the cavity via a loop located at the center of the top plate.

A microwave network analyzer was used to obtain necessary data for Q factor and resonant frequency measurements. In the presence of coupling losses, the method described by Malter and Brewer [4] was used for accurate measurement of the Q factors. The measurement of dielectric properties requires the measurement of resonant frequency, unloaded and loaded cavity Q factors and the sample height in the cavity. The sample height in the cavity varied between 1.5 cm and 4 cm. The theory and experimental apparatus has been previously described in detail [1].

The materials studied here have been used in the laboratory of the authors as heat transfer media and/or catalyst in fluidized bed reactors for conversion of biomass to clean energy fuels [5]. Microwave-heated fluidized bed reactors are potential alternatives to conventionally heated fluidized bed reactors. Therefore, the dielectric properties of the materials reported here were determined to understand their microwave energy absorbance at higher temperatures. The cobalt/alumina (Co/Al) catalyst was prepared in the laboratory. The dielectric properties of this catalyst were determined both in uncalcined and calcined state. Table I shows the uncalcined catalyst dielectric properties. Two samples were used. The uncalcined room temperature dried catalyst has a considerable amount of absorbed water ($Co(NO_3)_2 \cdot 6H_2O/Al_2O_3$). Therefore, it is expected to be lossy. However, when the uncalcined sample is dried at 82°C, the anhydrous form is obtained. This sample has very low loss as seen in the table.

The dielectric constant of five different samples as a function of temperature is shown in Fig. 3. The dielectric constant stays almost constant for all samples except for the Co/Al catalyst. The Co/Al catalyst was calcined in a microwave heated fluidized bed reactor prior to the measurement. The loss factors of Co/Al catalyst and the other four samples are shown in Figs. 4 and 5, respectively. As seen, the calcined Co/Al catalyst is considerably lossy. Micro-

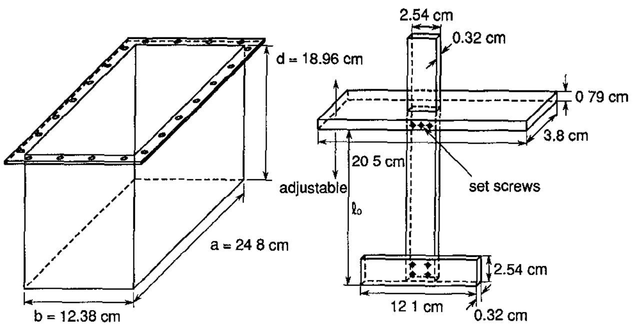

Fig. 2. Stainless steel rectangular cavity and sample leveling piece.

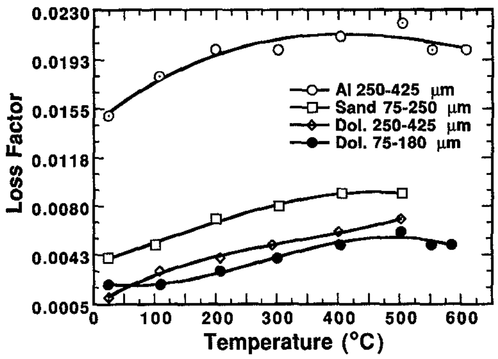

Fig. 5. Loss factor variation with temperature.

TABLE I
DIELECTRIC PROPERTIES OF UNCALCINED Co/Al CATALYST

Sample	Dielectric Constant (ϵ')	Loss Factor (ϵ'')
Dried at room temperature	2.96	0.283
Dried in oven at 82°C	2.68	0.024

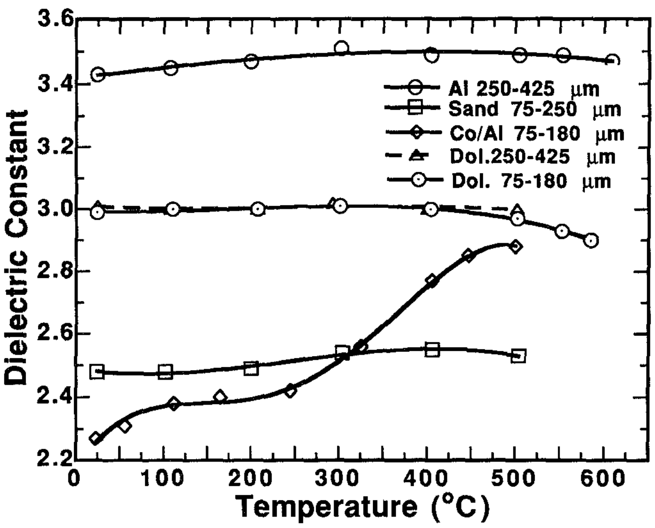

Fig. 3. Dielectric constant variation with temperature.

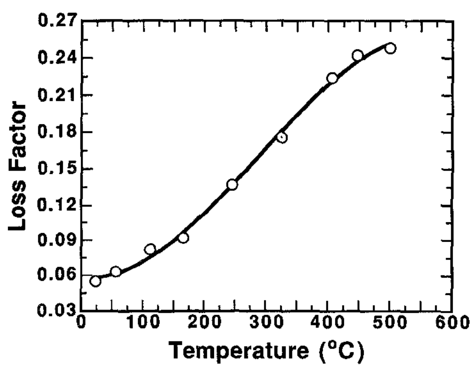

Fig. 4. Loss factor of Co/Al catalyst as a function of temperature.

wave calcination of this catalyst in a fluidized bed at 400°C proved to be rapid and without operational difficulties [6]. This confirmed its relatively lossy behavior under microwave energy. The loss factor of sand and dolomite samples were much smaller than that of alumina (Al_2O_3) and calcined Co/Al catalyst. The loss factor of these samples did not change appreciably within the temperature range studied. However, the loss factor Co/Al catalyst at 500°C increased to about five times its value at room temperature. The calcined Co/Al catalyst contained approximately 84% Al_2O_3 and 16% CoO. The loss factor of pure Al_2O_3 has been found to increase in the frequency and temperature range reported [7]. Furthermore, the conductivity of alumina also increases in the frequency and temperature range studied. The increase in conductivity is responsible for the increase in the loss factor. The second component in the catalyst (CoO) may also be contributing to the increase of conductivity

$$\left(\epsilon'' = f\left(\frac{\sigma}{\omega\epsilon_o}\right) \text{ where } \sigma \text{ is the conductivity}\right).$$

There, the loss factor increase is observed as the temperature of the catalyst is increased. The measurements were conducted under nitrogen atmosphere. Upon visual inspection of the catalyst after the measurements, no apparent phase change was observed.

The results presented here were corrected for temperature effects. Thermal expansion of the cavity significantly affected the results [6]. As the cavity expands, the resonant frequency decreases. The decrease, purely due to thermal expansion of the cavity, has to be taken into account in calculation. Corrections affected mostly the dielectric constant results while minimal effect on loss factor was observed.

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